Fitting Curves to Data

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The Case

- Suppose we want to project what the US population will be in 2010
- One approach is to fit past data to a curve and extrapolate

Census Data

date	Population (millions)
1900	75.995
1910	91.972
1920	105.711
1930	123.203
1940	131.669
1950	150.697
1960	179.323
1970	203.212
1980	226.505
1990	249.633
2000	281.422

The Curve

$$population = Ke^{\alpha t}$$

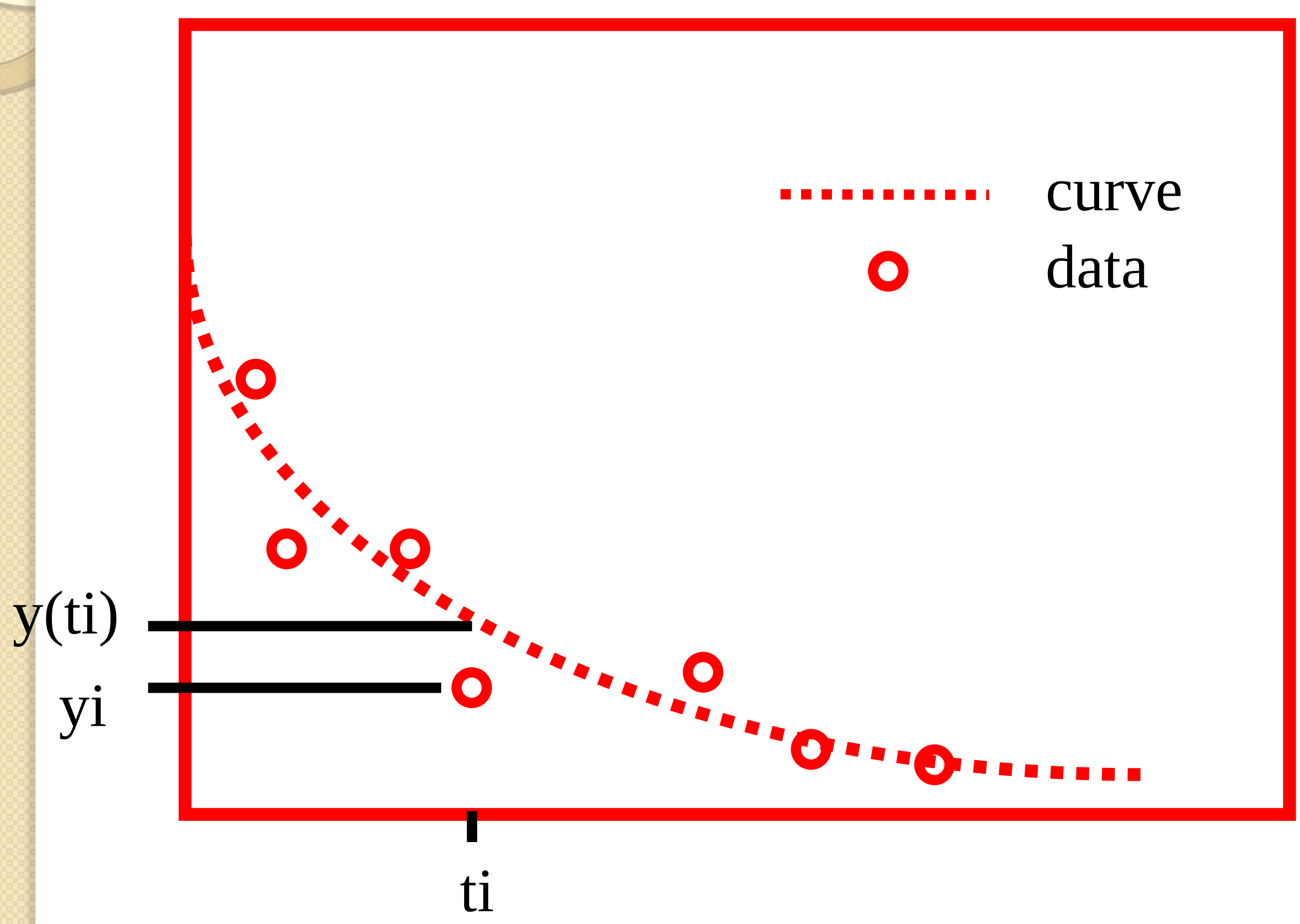
- Find K and α to achieve best fit

Fitting Curves to Data

- Generally curve fitting involves least-squares fits
- We seek parameters in a function that minimize the sum of the squares of the differences between curve and data

$$F = \sum_{i=1}^{N} \left[y_i - y(t_i; K, \alpha) \right]^2$$

Graphical Representation



Linear vs. Nonlinear

- Linear:

$$y = a + bt$$

$$y = a + bt + ct^2$$

$$y = a\sin(t) + b\cos(t)$$

$$y = a\sin(3t)$$

$$y = ae^{-t}$$

- Nonlinear:

$$y = a\sin(bt) + c\cos(dt)$$

$$y = a\sin(bt)$$

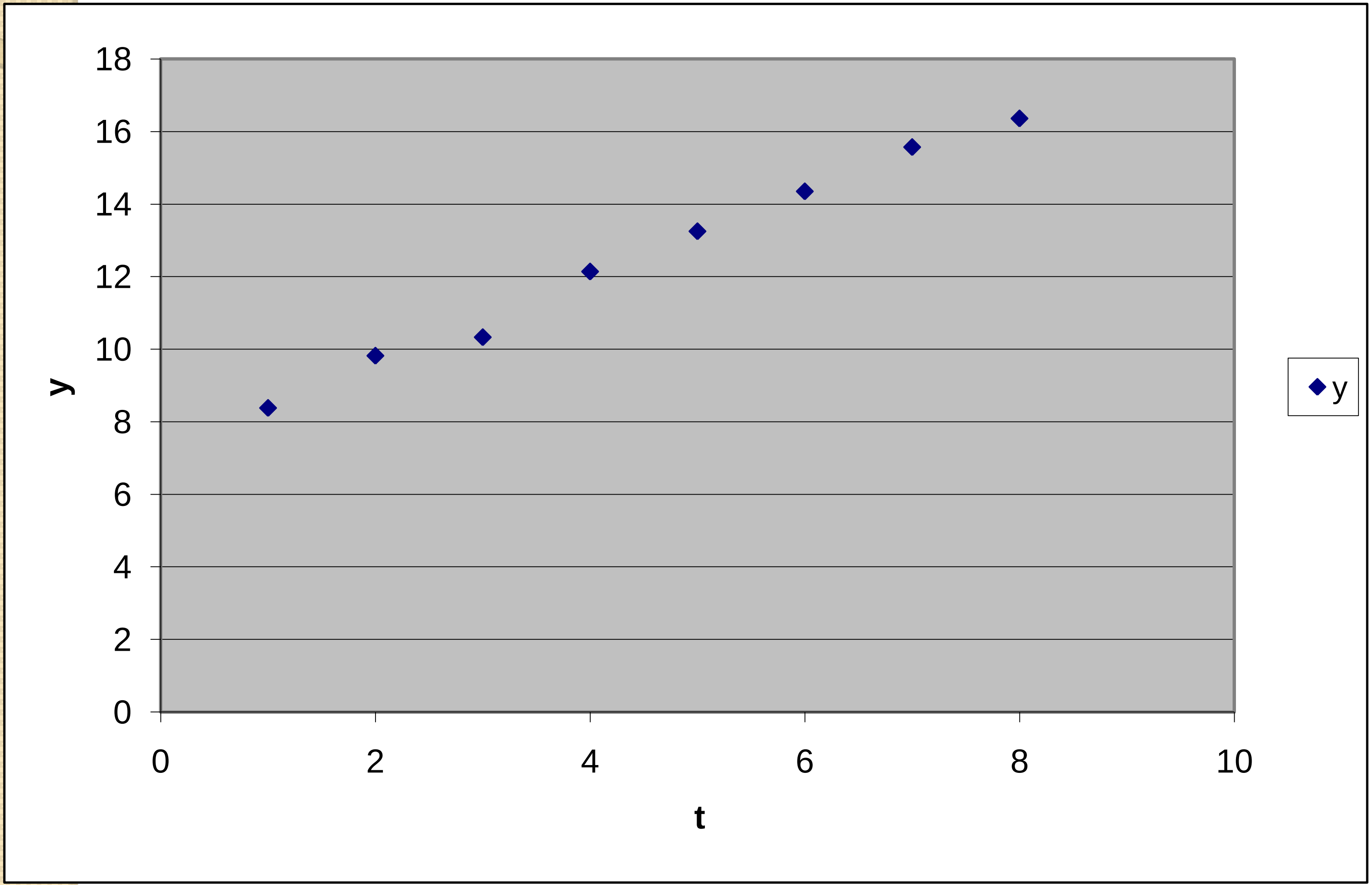
$$y = ae^{-bt}$$

Simple Example

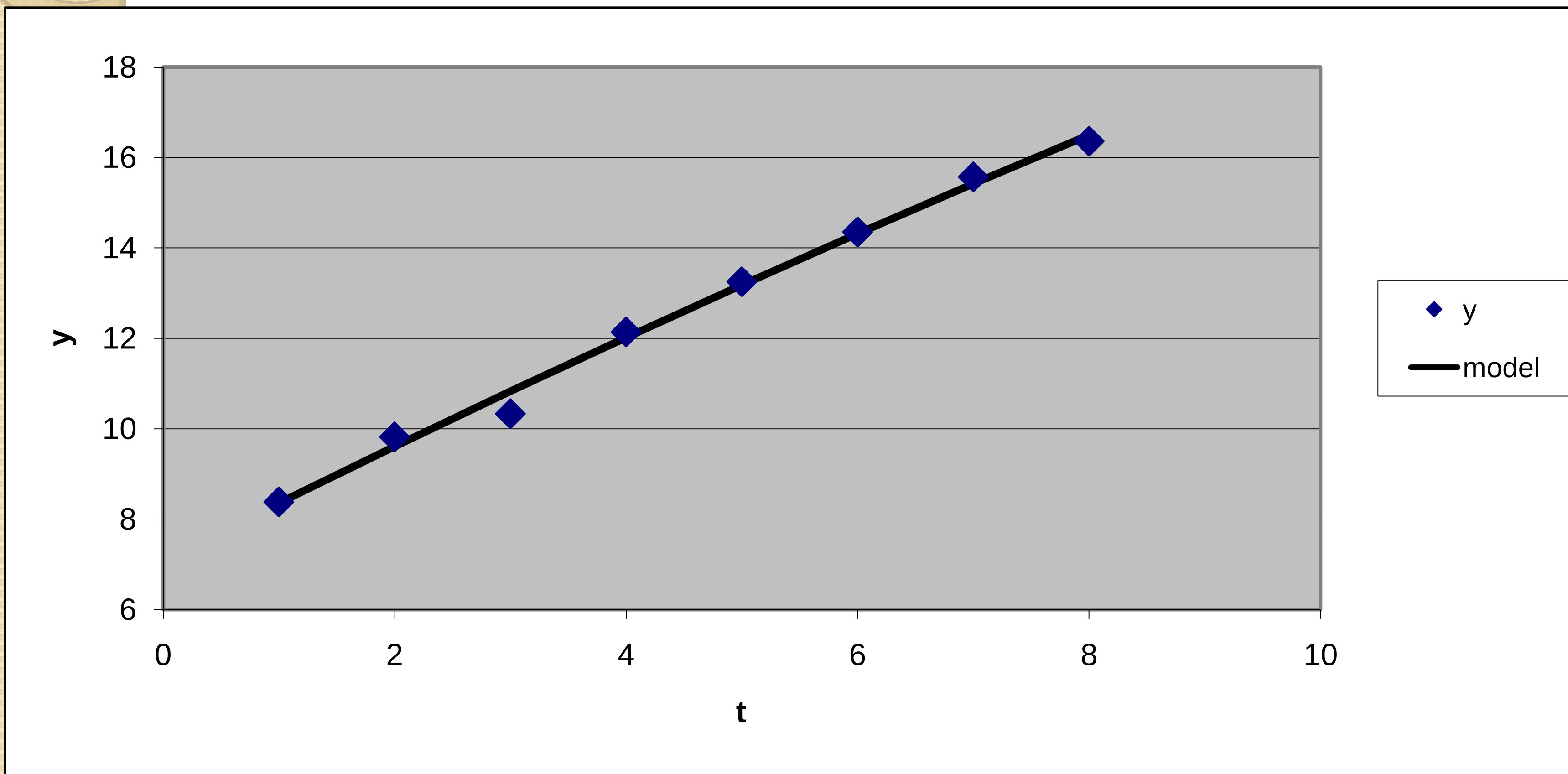
- Data:

t	y
1	8.38
2	9.82
3	10.33
4	12.14
5	13.25
6	14.35
7	15.57
8	16.36

Plot



Result



Matlab

- Use polyfit
- Fit from figure window
- fminsearch for nonlinear fits

Using polyfit in Matlab

- Polyfit fits a polynomial to a set of data
- Polyval allows evaluation of the resulting data in order to plot the results

Sample Commands (straight line)

```
tdata=1:5;
ydata=[8.38 9.82 10.33 12.14 13.25];
coefs=polyfit(tdata, ydata, 1)
t=1:0.1:5;
y=polyval(coefs,t);
plot(t,y,tdata,ydata,'o')
```

Demo of Interactive Fit

Practice

- Fit population data to straight line
- What will population be in 2010?
- Repeat for quadratic
- Repeat for cubic

Scaling the "x" data

- Fitting will work better if we "scale" the data
- Our goal is to get a set of x data with a mean of 0 and a standard deviation of 1
- Get this by calculating mean (μ) and std (σ) of the x data and then fit to z, where

$$z = \frac{x - \mu}{\sigma}$$

More on Scaling

- Wizard for fitting data will do this automatically

Nonlinear Fits

- Nonlinear fits are much more difficult
- There isn’t necessarily a unique solution to the problem
- We have to provide an initial guess for the parameters and then hope the tool can converge to a solution
- This is easily done with the Solver in Excel, but takes a bit more work with Matlab

What we need

- To carry out nonlinear fits, we need the following:
  - A function to evaluate the model for a given set of parameters and for a given time (this is the curve we are fitting to the data)
  - A function to calculate the sum of the squares of the errors between the model and the data (for a given set of fitting parameters)
  - A routine to put everything together

Nonlinear Fits in Matlab (Calling Script)

```
x=[1; 2; 3; 4; 5];
y=[0.9; 7.0; 28.3; 62.1; 122.4];
numpts=max(size(x));
zin(1)=1; %guess for first parameter
zin(2)=3; %guess for second parameter
zout=fminsearch(@(z) sumoferrs(z,x,y), zin)
xplot=x(1):(x(end)-x(1))/(10*numpts):x(end);
yplot=curve(xplot,zout);
plot(x,y,'+',xplot,yplot)
```

Curve for Nonlinear Fits

```
function f=curve(x,z)
a=z(1);
n=z(2);
f=a*x.^n;
```

Routine to Find Sum of Errors

```
function f=sumoferrs(z, x, y)
f=sum((curve(x,z)-y).^2);
```

Practice

- Fit population data to exponential
- What will population data be in 2010?
- Approach:
  - Download **nonlinfit.m**
  - Replace data (x and y) in this file with population data from **uspop.m**
  - Fix guesses for k and alpha - k=z(1) and alpha=z(2)
  - Change curve function to provide f=k*exp(alpha*t)

Questions?